excellent for those events initiated 0·4 ft from the bottom of the channel; however, the coupled solution under-predicts by 25 per cent the point of reentry for an event initiated 0·66 ft from the bottom.

The assumption of perfectly smooth interfaces between the hot plug of liquid pushed into the cold zone and the cold film is another simplification of the actual physics of the expulsion. The mixing of the hot and cold fluids as the hot fluid passes through the cold zone has been neglected. This mixing effect will be greatest when the velocity of the liquid slug being pushed into the cold zone is small. Thus, for a low superheat expulsion initiated high in the test section as in Fig. 4, the mixing effect will be the greatest, and the coupled solution will overpredict the rate of condensation within the cold zone.

The pressure within the vapor space for the event shown in Fig. 2, in which the initial superheat is high, drops rapidly to a value intermediate between the initial pressure and the plenum pressure (0·212 atm). Thus, the growth of the vapor space for this event is both energy and intertia controlled.

Figure 3 illustrates the pressure within the vapor space for events in which the vapor slug underwent a period of growth followed by a partial collapse and additional period of growth. The pressure increases slightly during the reentry phase. As the slugs are expelled again, the pressure follows a path similar to that observed during the initial expulsion.

Figure 4 illustrates the pressure within the vapor space for an event in which total collapse occurs. A pressure increase of 3–4 atm was recorded. After the vapor space collapsed, the slight increase in pressure corresponds to bubbles nucleating from the wire electrode. The coupled solution would predict a higher pressure pulse upon reentry than that indicated above. This is due to the fact that the coupled solution does not account for compressibility effects during the reentry phase.

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CORRELATION OF LIQUID-FILM COOLING MASS TRANSFER DATA

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NOTATION

e, fraction of m_1 entrained into gas flow;
e_0, value of e for $l = l_0 = 10$ in.;
$h_{l,m}$, liquid enthalpy at $T_{l,m}$;
$h_{l,1}$, liquid enthalpy at $T_{l,1}$;
$H_{v,m}$, liquid heat of vaporization at $T_{l,m}$;
l, liquid film length;
l_0, liquid film length of 10 in.;
m_1, rate of liquid injection per unit width of film;
m, net mass-transfer rate from liquid film per unit of film width;
m'_{id}, m' for ideal case where $r = e_0 = 0$;
r, roughness parameter;
T_g, free-stream gas temperature;
T_s, liquid film surface temperature;
$T_{l,1}$, liquid temperature at point of injection;

$T_{l,m}$, maximum film temperature (essentially the wet-bulb temperature);
u_g, free-stream gas velocity;
X_e, dimensional extrainment group, $= X_r/\sigma$;
X_r, dimensional roughness group, $= (\rho_g u_g^2)^{\frac{1}{2}}(T_g/T_s)^{\frac{1}{4}}$;
ρ_g, free-stream gas density;
σ, liquid surface tension.

INTRODUCTION

THE INTRODUCTION of a thin liquid-film onto a solid surface has frequently been employed as a means of protecting that surface from the thermal environment of a proximate hot gas stream. The wetted area which can be protected for a given rate of liquid injection is governed by the net rate of mass transfer resulting from the combined effects of liquid evaporation and mechanical entrainment of liquid droplets due to the interfacial shear. The results of a recent experimental study of the rate of evaporative and entrainment mass transfer from a liquid film to a hot turbulent gas stream have been presented in [1]. Extensive experimental data for a constant velocity gas flow over a liquid film of fixed length, $l_0 = 10$ in., resulted in the following empirical correlation for the net measured rate of mass transfer.

$$m' = (1 + r)m'_{id} + e_0 m_1 \tag{1}$$

where

$$r = \frac{3\cdot0}{X_r^{0\cdot5}} \tag{2}$$

and

$$e_0 = 1\cdot0 - \exp[-5 \times 10^{-5}(X_e - 1000)]. \tag{3}$$

The term m'_{id} in equation (1) represents the ideal mass-transfer rate for a 10-in. long, smooth, zero velocity, liquid film as predicted for the turbulent boundary-layer flow. The *roughness parameter* r in equation (1) accounts for the increase in convective mass transfer due to the effective roughness of the liquid film surface, while the *entrainment parameter* e_0 represents the fraction of the liquid injected onto the surface (m_1) that is entrained into the gas flow. The parameters r and e_0, evaluated from the experimental data, were correlated, as shown in equations (2) and (3), in terms of a dimensional *roughness group* X_r defined as

$$X_r = (\rho_g u_g^2)^{\frac{1}{2}}(T_g/T_s)^{\frac{1}{4}}[\mathrm{lb_f}^{\frac{1}{2}}/\mathrm{ft}] \tag{2a}$$

and a dimensional *entrainment group* X_e defined by

$$X_e = \frac{(\rho_g u_g^2)^{\frac{1}{2}}}{\sigma}(T_g/T_s)^{\frac{1}{4}}[\mathrm{lb_f}^{-\frac{1}{2}}]. \tag{3a}$$

The purpose of this note is to extend the above correlation to liquid films of arbitrary length, l, to permit the correlation of data reported by Kinney, Abramson and Sloop [2] and Emmons and Warner [3] for liquid-film cooling with films ranging in length from 3 to 36 in. The extension of this relatively simple correlation to flow conditions significantly different from those investigated in [1] is illustrated by the comparison of the nominal experimental parameters shown in Table 1.

ANALYSIS

A correlation for the general case of mass transfer from a liquid form of arbitrary length l is suggested by equation (1) modified to the form

$$m' = (1 + r)\, m'_{id} + e\, m_1. \tag{4}$$

The entrainment parameter e in equation (4), applicable for the liquid film of length l, is expected to be a function of l/l_0, in addition to X_e, such that $e = e_0$ when $l/l_0 = 1$.

The determination of the functional dependence of e on l/l_0 can be simplified by examining the case where a liquid film, injected at a rate m_1, terminates naturally due to the mass transfer over the length l. Since $m' = m_1$ in that case, equation (4) becomes

$$m_1 = (1 + r)\, m'_{id}(1 - e)^{-1}. \tag{5}$$

Table 1. Comparison of nominal flow parameters

Parameter	Gater and L'Ecuyer [1]	Kinney, Abramson and Sloop [2]	Emmons and Warner [3]
T_g (F)	40–600	800–2000	3640
p (psia)	75–150	23–37	500
u_g (fps)	34–430	1000–1700	38–127
$\rho_g u_g$ ($\mathrm{lb_m/ft^2\ s}$)	12–93	39–82	10–33
$\rho_g u_g^2$ ($\mathrm{lb_f/ft^2}$)	20–1087	700–3000	12–130
X_e ($\mathrm{lb_f^{-\frac{1}{2}}}$)	1570–19790	6700–13700	1600–7300
X_r ($\mathrm{lb_f^{-\frac{1}{2}}/ft}$)	5–38	36–70	5–18
l (in.)	10	9–36	3–8
liquids	water methanol butanol RP-1	water	water ethanol ammonia Freon 113

An analysis to predict the dependence of e on l/l_0 has been developed [4] by considering the liquid film to be made up of a number of elements of length l_0 and successively applying equation (1) to determine the mass transfer from each film element. The resultant analytical expression giving $(1 - e)$ as a series expansion in terms of $(1 - e_0)$ can be approximated by the more convenient empirical relation

$$(1 - e) = (1 - e_0)^{(l/l_0)^b}. \tag{6}$$

A value of $b = 0.5$–0.6 was found to yield a good approximation to the series expansion for $l/l_0 = 1$–4. Thus, the relationship between coolant injection flowrate and film cooled length is given by

$$m_l = m'_{id}\left[1 + \frac{3.0}{X_r^{0.8}}\right]\exp\left[5 \times 10^{-5} \times (X_e - 1000)(l/l_0)^b\right]. \tag{7*}$$

In the present study, equation (7) was employed to compute l as a function of m_l for comparison with the experimental data of [2] and [3]. In the analysis, a constant velocity, turbulent boundary layer flow was assumed in evaluating m'_{id}, neglecting the effects of radiation heat transfer and chemical reactions. The evaluation of the surface tension, σ, in the entrainment group, X_e, was based on a reference temperature This was necessitated since in the experiments of [2] and [3], the liquid coolant was injected at essentially the ambient temperature which resulted in a substantial change in the liquid temperature (and therefore σ) over the length of the wetted test section. (This problem was particularly acute for the investigation of [3].) The reference temperature was defined as

$$T_r = wT_{l,m} + (1 - w)T_{l,1} \tag{8}$$

where

$$w = \frac{H_{v,m}}{H_{v,m} + h_{l,m} - h_{l,1}}. \tag{9}$$

CORRELATION OF THE DATA

The data of Kinney, Abramson and Sloop

Data reported for water film cooling of 2-in. and two 4-in. dia. ducts are compared with predictions from equation (7). The 2-in. dia. duct and one of the 4-in. dia. ducts were seamless tubes having a honed inner surface, and are termed the smooth-surface ducts. The second 4-in. dia. duct, termed the rough-surface duct, was a rolled-tube with a longitudinal weld. Figure 1 represents the data for the smooth-surface ducts in terms of the film cooled length versus the injected liquid flow rate. Also shown in the figure are the corresponding results computed from both the ideal theory of mass transfer (i.e. no film roughness or entrainment) and

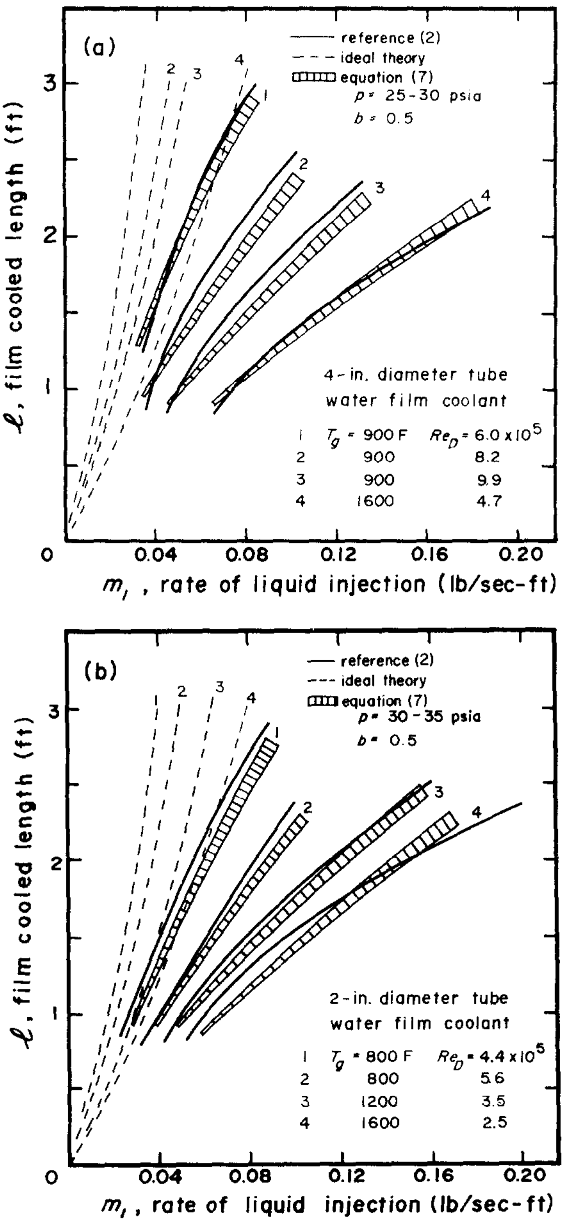

FIG. 1. Analysis of the data of Kinney, Abramson and Sloop for liquid-film cooling smooth-surface ducts. (a) 4-in. dia. (b) 2-in. dia.

*Although equation (6) was developed for $l/l_0 > 1$, the analysis of the data of [3] presented herein indicates the utility of equation (7) for $l/l_0 < 1.0$. However, the applicability of equation (7) for very short films ($l \lesssim 0.3\ l_0$) has not been demonstrated.

equation (7). The calculations for the present study are based on a value of 0·5 for the parameter b in equation (7) to provide the best fit of the data. (A value of $b = 0{\cdot}6$ resulted in mass transfer rates generally 10 per cent greater than those indicated.) Figure 1 clearly illustrates the influence of film roughness and entrainment on the mass-transfer rate. For example, Fig. 1 (a) shows for a gas stream Reynolds number of $4{\cdot}7 \times 10^5$ and a gas stream temperature of 1600F, that the value of m_l required to establish a film cooled length of 2·2 ft is more than three times the value predicted for the ideal case. For the subject flow conditions, entrainment mass transfer accounts for the bulk of this difference.

Figure 2 presents a similar comparison of the experimental results for the rough-surface tube and values calculated from equation (7). A value of $b = 0{\cdot}6$ was employed to obtain the best fit of the data. In general, the agreement between the experimental data and the results calculated from the empirical correlation extended herein is satisfactory, with the greatest difference occurring for the experimental data at the larger rates of liquid injection.

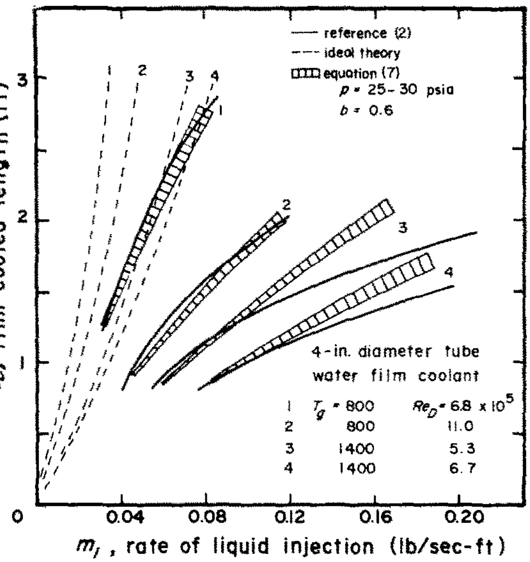

FIG. 2. Analysis of the data of Kinney, Abramson and Sloop for liquid-film cooling of a 4-in. dia, rough-surface duct.

The data of Emmons and Warner

Experiments were conducted with a film cooled, 3·14-in. dia. rocket motor combustion chamber to determine the effects of gas stream temperature, pressure, and Reynolds number on the flow rate of water required to establish a range of film cooled lengths (l = 4–8 in.). Experiments also were conducted with ethanol, ammonia, and Freon-113 at one gas stream flow condition to study the influence of the physical properties of the liquid on the rate of mass transfer. Most of the reported data are for a gas stream temperature of 4100 R and a chamber pressure of 500 psia. Only those data are considered here.

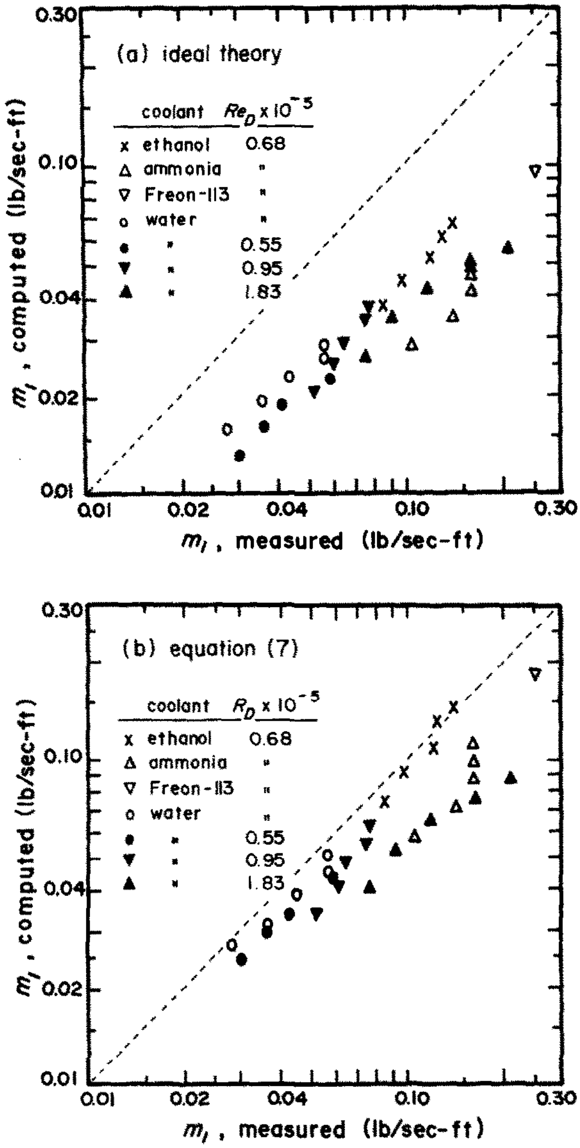

FIG. 3. Analysis of the data of Emmons and Warner for liquid-film cooling a small rocket motor. (a) The ideal theory. (b) Equation (7).

Figure 3 shows a comparison of the computed results with the experimental data. Figure 3 (a) presents the rate of mass transfer predicted from the ideal theory versus the measured rate of mass transfer, while Fig. 3 (b) gives a similar comparison of the experimental data and the results computed from equation (7) ($b = 0{\cdot}6$). Figure 3 shows that while the empirical correlation proposed herein predicts a mass-transfer rate that is consistently less than that realized experimentally, it represents a substantial improvement over that obtainable from the ideal theory. Indeed, if radiation heat transfer had been accounted for it is likely the agreement between the computed results and the experimental data would have been considerably improved.

CONCLUSIONS

The empirical correlation proposed in [1] for liquid-film cooling mass transfer, accounting for film roughness and entrainment effects, is extended to include liquid films of arbitrary length. A favorable comparison between the predicted results and the experimental data of Kinney, Abramson and Sloop [2] and Emmons and Warner [3] demonstrates the utility of the mass transfer correlation for predictions over a considerably wider range of experimental parameters than those investigated in [1].

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THE CRITERION FOR VALIDITY OF THE FIN APPROXIMATION

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NOMENCLATURE

- A, cross-section area of fin;
- h, longitudinal surface convective coefficient;
- h_e, end surface convective coefficient;
- H, dimensionless fin parameter mh_e/h;
- J_0, J_1, ordinary Bessel functions of zeroth and first order;
- k, thermal conductivity of fin material;
- m, dimensionless fin parameter $(hP/kA)^{\frac{1}{2}}$;
- P, perimeter of fin cross-section area;
- r', radial coordinate;
- R', outer radius of fin;
- x', y', transverse fin coordinates;
- z', longitudinal or axial fin coordinate;
- T_f, surrounding fluid temperature;
- T_w, base temperature of fin;
- θ, dimensionless temperature $(T - T_f)/(T_w - T_f)$.

INTRODUCTION

The analysis of temperature distribution and heat flux in fins customarily makes use of a one-dimensional fin approximation wherein temperature gradients in the direction normal to the convective surface are neglected. Many texts and references which discuss this topic [1–5], some recent, state that the fin approximation is valid when transverse dimensions are small compared to the length of the fin. Crank and Parker [6] have shown, however, that in thin plates with convective surfaces, temperature gradients in the